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People's Democratic Republic of Algeria  
Ministry of Higher Education and Scientific  
Research  
University of Saida Dr. Tahar MOULAY

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Laboratory of Stochastic Models, Statistic and Applications.

# The First International Day on Probabilities

(Online)

Saida on March, 16<sup>th</sup> 2022

The 11<sup>th</sup> Anniversary of LMSSA

and in Tribute to The Memory of Professors

Benamar CHOUAF And Tahar MOURID

international day on probabilities - LMSSA  
March, 16<sup>th</sup> 2022

Alexandre Dumas said:

"Ceux que nous avons aimés et que nous avons perdus ne sont plus où ils étaient, mais ils sont toujours et partout où nous sommes."

Professors..... You will be forever attached to our memories.

May your souls rest in peace.

To the memory of professors

Benamar CHOUAF And Tahar MOURID.

The First(Online) International Day on Probabilities  
the web page:  
*\https://www.univ – saida.dz/jiprob1/*

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# I. Plenary Lectures

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## Solving Mean-field Stochastic Control Problems by Using Deep Learning.

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**Abstract :** The two famous approaches of solving stochastic control problems are Bellman's dynamic programming and Pontryagin's maximum principle. The dynamic programming method can be very efficient, but it works only if the system is Markov. The maximum principle, on the other hand, does not require that the system is Markov, but it has the drawback that it involves complicated backward stochastic differential equations. The mean-field systems are not Markovian a priori, but they can be made Markovian by adding to the system the Fokker-Planck equation for the law of the state. Then we can use the dynamic programming to study optimal control of mean-field equations.

Mean-field dynamics have a lot of applications, in this talk I will represent in particular two applications: Optimal energy consumption by the cortex neural network and initial investment problems. We will apply stochastic control methods to solve the problems. Furthermore, it is sometimes difficult to find explicit solutions mathematically and therefore, we will use numerical method to find them.

We will use deep learning techniques to solve special cases of the above discussed problems explicitly.



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## Le processus de Rosenblatt

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**Résumé:** Dans cet exposé, je présenterai des résultats récents et plus anciens concernant le processus de Rosenblatt, qui est une généralisation non gaussienne du mouvement brownien fractionnaire.

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## Fokker-Planck PIDE for McKean-Vlasov Diffusions with Jumps, and Applications to HJB Equations and Optimal Control.

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**Abstract :** We study optimal control of McKean-Vlasov (mean-field) stochastic differential equations with jumps.

- First we prove a Fokker-Planck equation for the law of the state.
- Combining this equation with the original state equation, we obtain a joint Markovian system for the state and its law.
- We apply this to formulate a Hamilton-Jacobi-Bellman (HJB) verification equation for the optimal control of McKean-Vlasov stochastic equations with jumps.
- Finally we apply these results to solve explicitly the following problems:
  - (i) Linear-quadratic optimal control of stochastic McKean-Vlasov equations with jumps;
  - (ii) Optimal consumption from a cash flow modelled as a stochastic McKean-Vlasov equation with jumps.

The presentation is based on joint work with Nacira Agram, Linnaeus University (LNU) and Royal Institute of Technology (KTH), Sweden.

## II. Oral Communications

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**Existence of an Almost Periodic Solution for Class of Stochastic  
Differential Equations Driven by Fractional Brownian Motion.**

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**Abstract** This work deals with existence and uniqueness of almost periodic solutions in distribution for a class of stochastic differential equations driven by a fractional Brownian motion with almost periodic coefficients

By using the chaos decomposition approach and the representation of the fractional Brownian motion in terms of a standard Brownian motion, we show the existence and uniqueness of almost periodic solution in distribution to affine stochastic differential equation driven by a fractional Brownian motion with Hurst parameter  $H \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$ , the stochastic integral considered is of the divergence type of Malliavin calculus. [4]

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## An Anticipating Stochastic Integral with Respect to the Mixed Fractional Brownian Motion.

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**Abstract :** In this paper, we define a stochastic integral of an anticipating integrand with respect to a mixed fractional Brownian motion based on Ayed and Kuo's approach [1]. This provides a new concept of stochastic integration of non-adapted processes. In addition, under some conditions, we prove that our anticipating integral is a near-martingale.

**keywords:** instantly independent process, non-adapted process, mixed fractional Brownian motion, anticipating integral.

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### The Stochastic Flow on Manifolds in a Multidimensional Case.

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**Abstract** We are interested essentially to the so-called natural model, this model is expressed by a stochastic differential equation called  $\natural$ -equation. In this work, we try to demonstrate one of the properties of the stochastic flow generated by this model, more precisely we demonstrate the property of homeomorphism based on Hiroshi Kunita's theory.

**subjclass[2010]** Primary 60G17; Secondary 60H05 **keywords** Credit risk; Stochastic flow; Stochastic differential geometry; Diffeomorphism

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## Fractional Stochastic Evolution Equations In a The Hilbert Space.

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**Abstract:** Our goal is to prove the existence of square-mean piecewise almost periodic solutions of the following impulsive fractional stochastic differential equations:

$$\begin{cases} {}^c D_t^\alpha x(t) + Ax(t) = F(t, x(t)) + \Sigma(t, x(t)) \frac{dW(t)}{dt} + \sum_{k=-\infty}^{\infty} G_k(x(t)) \delta(t - \tau_k), t \in J = \mathbb{R}, \\ x(t_0) = x_0. \end{cases} \quad (1)$$

where the state  $x(\cdot)$  takes values in the space  $L_2(\Omega, \mathcal{H})$ ,  $\mathcal{H}$  is a separable real Hilbert space with inner product  $(\cdot, \cdot)$  and norm  $\|\cdot\|$ ; the fractional derivative  ${}^c D^\alpha$ ,  $\alpha \in (0, 1)$ , is understood in the Caputo sense;  $-A : \mathcal{D}(A) \subset L_2(\Omega, \mathcal{H}) \rightarrow L_2(\Omega, \mathcal{H})$  is the infinitesimal generator of an analytic semigroup of a bounded linear operator  $S(t)$ ,  $t \geq 0$ , on  $L_2(\Omega, \mathcal{H})$  satisfying the exponential stability;  $\{W(t) : t \geq 0\}$  is a given  $\mathcal{K}$ -valued Wiener process with a finite trace nuclear covariance operator  $Q \geq 0$  defined on a filtered complete probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ ,  $\mathcal{K}$  is another separable Hilbert space with inner product  $(\cdot, \cdot)_{\mathcal{K}}$  and norm  $\|\cdot\|_{\mathcal{K}}$ ;  $G_k : \mathcal{D}(G_k) \subset L_2(\Omega, \mathcal{H}) \rightarrow L_2(\Omega, \mathcal{H})$  are continuous impulsive operators,  $\delta(\cdot)$  is Dirac's delta-function,  $F(t, x) : \mathbb{R} \times L_2(\Omega, \mathcal{H}) \rightarrow L_2(\Omega, \mathcal{H})$  and  $\Sigma(t, x) : \mathbb{R} \times L_2(\Omega, \mathcal{H}) \rightarrow L_2(\Omega, L_0^2(\mathcal{K}, \mathcal{H}))$  are jointly continuous functions (here,  $L_0^2(\mathcal{K}, \mathcal{H})$  denotes the space of all  $Q$ -Hilbert-Schmidt operators from  $\mathcal{K}$  into  $\mathcal{H}$ ).

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## Stochastic Differential Inclusion with Hilfer Fractional derivative.

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**Abstract** We study the existence of mild solution of Hilfer fractional stochastic differential inclusion driven by sub fractional Brownian motion in the cases when the multivalued map is convex and non convex. The results are obtained by using fixed point theorem. Finally an example is given to illustrate the obtained results.

**Key words and phrases:** Hilfer fractional derivative, fixed point, stochastic differential inclusion, sub-fractional Brownian motion, mild solution.

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## The Numerical Solution of Singular Stochastic Differential Equations.

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**Abstract:** In this paper we are interested to solve numerically quadratic SDEs with non-necessary continuous drift of the form

$$X_t = x + \int_0^t b(s, X_s) ds + \int_0^t f(X_s) \sigma^2(X_s) ds + \int_0^t \sigma(X_s) dW_s, \quad (2)$$

where,  $x$  is the initial data  $b$  and  $\sigma$  are given coefficients that are assumed to be Lipschitz and bounded and  $f$  is a measurable bounded and integrable function on the whole space  $\mathbb{R}$ .

Numerical simulations for this class of SDE of quadratic growth and measurable drift, induced by the singular term  $f(x)\sigma^2(x)$ , is implemented and illustrated by some examples. The main idea is to use a phase space transformation to transform our initial SDEs to a standard SDE without the discontinuous and quadratic term. The Euler–Maruyama scheme will be used to discretize the new equation, thus numerical approximation of the original equation is given by taking the inverse of the space transformation. The rate of convergence are shown to be of order  $\frac{1}{2}$ .

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## Approximation and Stability Results In Relaxed Control Problems to G-Stochastic Functional differential equations

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**Abstract :** In the G-framework, we study the existence of an optimal relaxed control for stochastic functional differential equations driven by G-Brownian motion (G-SFDEs) with uncontrolled diffusion coefficient. The purpose of this works to study optimal control of systems that have model uncertainty or ambiguity due to inaccurate information, or vague concepts.

**Key Words:** stochastic functional differential equations, G-Brownian motion, Relaxed optimal control.

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**Stability with Respect to a Part of The Variables of Stochastic  
Differential Equations with a General Decay rate.**

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**abstract:** In this talk, practical stability with respect to a part of the variables of nonlinear stochastic differential equations is studied. The analysis of the global practical uniform asymptotic stability, the global practical uniform  $p$ th moment exponential stability, as well as the global practical uniform exponential stability with respect to a part of the variables of SDEs are carried out by using the Lyapunov techniques. Further, we investigate the almost sure practical stability with respect to a part of the variables of stochastic differential equations with general decay rate. Some illustrative examples to show the usefulness of the stability with respect to a part of variables notion are also provided.

**Keywords:** Stochastic systems, Lyapunov techniques, Itô formula, Brownian motion, nontrivial solution, practical stability with respect to a part of the variables, Decay function.

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## Analysis of a Queueing System with Variant Vacation, Bernoulli Feedback, Balking and Server's States-dependent Reneging

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**Key words:** Queueing models, Vacation, Impatience, Bernoulli feedback, Simulation. **Abstract** We consider a single server Markovian feedback queue with variant of multiple vacation policy, balking, server's states-dependent reneging, and retention of reneged customers. We obtain the steady-state solution of the considered queue based on the use of probability generating functions. Then, the closed-form expressions of different system characteristics are derived. Finally, we present some numerical results in order to show the impact of the parameters of impatience timers on the performance measures of the system.

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## Non-densely Defined Fractional Stochastic Evolution Equations Driven by Fractional Brownian Motion.

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**Abstract:** In this paper we study a class of non densely defined fractional stochastic differential equation with non-instantaneous impulses driven by fractional Brownian motion under some conditions to proving existence and unicity of integral solution by using approximation methods.

**Key words:** Fractional derivative, stochastic functional differential equation, non densely defined operator, fractional Brownian motion.

**MSC 2010 No.:** 26A33, 34A37, 34A60, 34K40, 35R11, 60H15

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The First International Day on Probabilities,  
IDP'2022, 16<sup>th</sup> March, Saida, Algeria



## Optimal Consumption and Investment for Exponential Utility Function.

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### Abstract :

We investigate an optimal consumption and investment problem for Black-Scholes type financial market on the whole investment interval  $[0; T]$ . We formulate various utility maximization problem, which can be solved explicitly. The method of solution uses the convex dual function (Legendre transform) of the utility function. Related to this concept, we introduce and study the convex dual of the value function for our problem.

**Keywords:** Portfolio optimization; consumption; exponential utility; convex duality.

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## Fractional Stochastic Differential Equations Driven by G-Lévy Processes.

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**Abstract** In this work, we study the Fractional stochastic differential equations driven by G-Lévy processes, we discuss the existence and uniqueness of solutions. Under some assumptions we present the averaging principle of this type of equations

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## The Adomian Decomposition Method for Random Fractional Differential Equations.

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**Abstract :** In this research, we investigate the approximate solution of the random fractional differential equations using Adomian decomposition method. The convergence of the series obtained by this method are discussed.

**Key Words:** .Random differential equation, fractional differential equation, Adomian decomposition method

**MSC 2010:** 30C45, 39B72, 39B82.

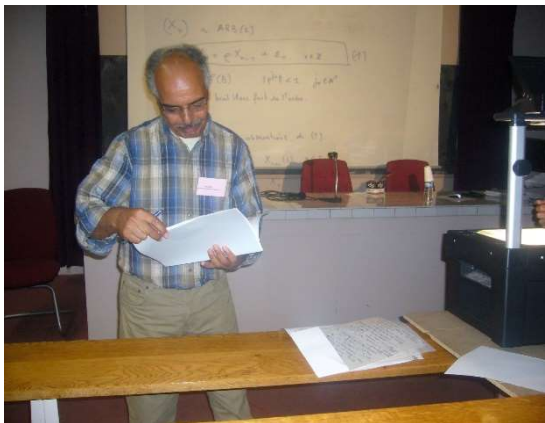
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This event has been organized for the first time by the Stochastic Models, Statistics and Applications Laboratory in order to celebrate, enrich and share knowledge on probability science (The stochastic differential equations driven by non-Gaussian processes and their applications, SDE associated with fractional operators and the use of process in financial mathematics). This day represents an opportunity for the researchers, and for doctorate students to take a look on the obtained results in this field and to have an exciting discussion on the new research and perspectives on the day theme.

16th 2022



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